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USE OF MODIFIED RELATIVE PERMEABILITIES FOR HYDRODYNAMIC CALCULATIONS IN STRATIFIED OIL BEDS

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The possibility of decreasing the dimensions of the problem of two-phase filtration in stratified beds through the introduction of modified phase permeabilities instead of the initial relative permeabilities (which are the coefficients of the initial system of equations within the framework of the Buckley–Leverett model) is investigated. Consideration is given to the physical and mathematical possibilities of constructing the modified permeabilities for the case where the relative permeabilities of each phase are presented by different analytical dependences for individual seams.

To model the flow of a multiphase fluid in oil beds (stratified in absolute permeability) without taking account of capillary forces we have proposed [1] generalized modified phase permeabilities. In the case of two-phase filtration they have the form

$$K_{\rm w}^{\rm mod}(S) = K_{\rm w}(S) A(S) \,, \quad K_{\rm oil}^{\rm mod}(S) = K_{\rm oil}(S) \, B(S) \,; \tag{1}$$

there $K_{\rm w}(S)$ and $K_{\rm oil}(S)$ are the relative permeabilities which are the coefficients of the initial system of differential equations within the framework of the Buckley–Leverett model. The correction factors $A(S)$ and $B(S)$ are obtained from the assumption of the jet character of displacement in the bed. In each vertical section, there are two zones: the zone of water ($S = S^*$) and the zone of oil ($S = S_*$). The factors $A(S)$ and $B(S)$ are found from the formulas

$$\overline{K}_{\rm w} = \frac{\int_{\overline{k}}^{b} kf(k)\, dk}{\int_{\overline{k}}^{b} f(k)\, dk} \,, \quad \overline{K}_{\rm oil} = \frac{\int_{a}^{\overline{k}} kf(k)\, dk}{\int_{a}^{\overline{k}} f(k)\, dk} \,, \quad K^* = \int_a^b kf(k)\, dk \,, \quad A(S) = \frac{\overline{K}_{\rm w}}{K^*} \,, \quad B(S) = \frac{\overline{K}_{\rm oil}}{K^*} \,.$$

In specific calculations, instead of these formulas one uses their discrete analogs. The quantity $\overline{k}$ is determined numerically from a known water saturation S from the equation

$$1 - S_{\rm mov}(S) = \int_a^{\overline{k}} f(k)\, dk \,, \quad S_{\rm mov}(S) = (S - S_*)/(S^* - S_*) \,.$$

Here $f(k)$ is the distribution density of the absolute permeability $K(z)$ varying from a to b over the bed thickness.

By introducing the modified permeabilities and the absolute permeability K^* averaged over the bed thickness we passed from the stratified bed with the permeabilities $K(z)$, $K_{\rm w}(S)$, and $K_{\rm oil}(S)$ to a fictitious homogeneous bed with the permeabilities K^*, $K_{\rm w}^{\rm mod}(S)$, and $K_{\rm oil}^{\rm mod}(S)$. The numerical calculations carried out in the two-dimensional and one-dimensional cases yielded a satisfactory proximity of the results of computations of the basic indices of development. Thus we were able to decrease the dimensions of the initial problem of two-phase filtration.

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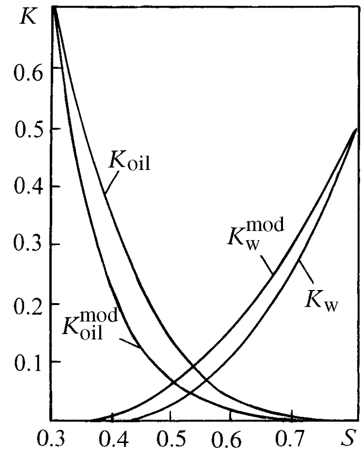


Fig. 1. Distribution of the absolute permeability in seams according to the Maxwell law. V = 0.84; $K_w(S)$ and $K_{oil}(S)$ are the cubic functions. K, D; S, dimensionless.

Figure 1 gives as an example the plots of the initial permeabilities $K_w(S)$ and $K_{oil}(S)$ and of modified initial permeabilities for cubic dependences on S for the Maxwell law of distribution with the coefficient of variation of a stratified inhomogeneity $V = 0.84$.

The modified permeabilities of water are always higher than the initial $K_w(S)$, while the modified permeabilities of oil are lower than the initial $K_{oil}(S)$. This is characteristic of any law of distribution by virtue of the range of variation of the factors $A(S)$ and $B(S)$. It is clear that $\overline{K}_w \geq K^*$ and $\overline{K}_{oil} \leq K^*$; therefore, $A(S) \geq 1$ and $0 \leq B(S) \leq 1$.

It is well known that the permeabilities $K_w(S)$ and $K_{oil}(S)$ often have different analytical dependences for different seams. It is impossible to construct modified permeabilities, equal for the entire thickness of the bed, of the form (1) by averaging the Darcy law [1]. Let us consider the case where the permeabilities are specified as

$$K_w(S) = K_{w0}\,[S_{mov}(S)]^{\alpha_i}, \quad K_{oil}(S) = K_{oil0}\,[1 - S_{mov}(S)]^{\beta_i}. \tag{2}$$

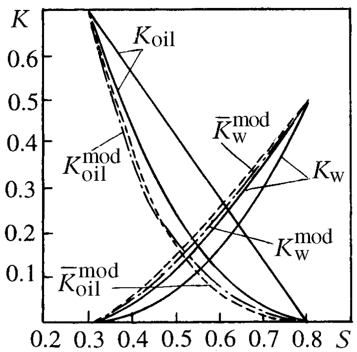
Here α_i and β_i depend on the number i of the seam (α_i and $\beta_i \geq 1$). We construct a modified permeability of water. To do this we pass to a fictitious stratified bed with a single permeability for all seams; we take this permeability in the form

$$K_w^*(S) = K_{w0}\,[S_{mov}(S)]^{\alpha^*}, \quad \alpha^* = \max \alpha_i. \tag{3}$$

This can be done from the following considerations. Modified permeabilities of water are always higher than the initial $K_w(S)$ under the assumption of jet flow. Therefore, we replace each homogeneous seam of the initial stratified bed for $\alpha_i < \alpha^*$ by a fictitious stratified seam with a certain complex of seams which are different in absolute permeability but have a single $K_w(S)$ of the form (3) and the average absolute permeability coincident with the absolute permeability of the ith seam. This fictitious stratified seam corresponds to a modified $K_w^{mod}(S)$ of the form (1); this modified permeability must coincide with the initial $K_w(S)$ (2) for the ith seam in question. We carry out the analogous constructions for the oil phase and obtain another fictitious seam with a single $K_{oil}^*(S)$. The obtained permeabilities have the form

$$K_w^{mod}(S) = K_{w0}\,[S_{mov}(S)]^{\alpha^*} A^*(S), \quad K_{oil}^{mod}(S) = K_{oil0}\,[1 - S_{mov}(S)]^{\beta^*} B^*(S). \tag{4}$$

Here $\beta^* = \min \beta_i$ and $A^*(S)$ and $B^*(S)$ are determined from the assumption of the jet nature of the flow in the first and second fictitious stratified beds respectively and are single for the initial stratified bed. The idea of such a construction is given in [2].



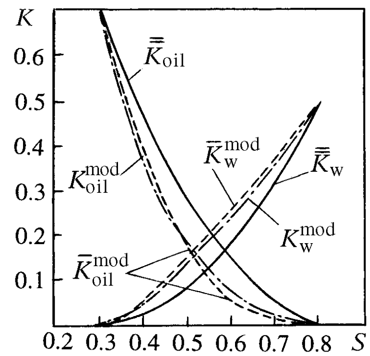


Fig. 2. Experimental permeabilities $K_w(S)$ and $K_{oil}(S)$ obtained in cores; modified permeabilities $K_w^{mod}(S)$ and $K_{oil}^{mod}(S)$ of the form (4) and modified permeabilities $\overline{K}_w^{mod}(S)$ and $\overline{K}_{oil}^{mod}(S)$ of the form (5).

Fig. 3. Modified permeabilities $K_w^{mod}(S)$ and $K_{oil}^{mod}(S)$ of the form (4); modified permeabilities $\overline{K}_w^{mod}(S)$ and $\overline{K}_{oil}^{mod}(S)$ of the form (5) and the permeabilities of the water and the oil $\overline{\overline{K}}_w(S)$ and $\overline{\overline{K}}_{oil}(S)$ averaged over the seam thickness.

Let us consider as an example a bed consisting of five seams of the same thickness with different initial laboratory relative permeabilities of the form

$$K_1 = 0.1\text{ D},\quad K_2 = 0.3\text{ D},\quad K_3 = 0.5\text{ D},\quad K_4 = 0.7\text{ D},\quad K_5 = 0.9\text{ D};$$

$$K_w(S) = K_{w0}\,[S_{mov}(S)]^2 \quad \text{for} \quad i = 1, 2, 4, 5,\quad K_w(S) = K_{w0}\,[S_{mov}(S)]^{1.5} \quad \text{for} \quad i = 3;$$

$$K_{oil}(S) = K_{oil0}\,[1 - S_{mov}(S)]^2 \quad \text{for} \quad i = 1, 2, 4, 5,\quad K_{oil}(S) = K_{oil0}\,[1 - S_{mov}(S)] \quad \text{for} \quad i = 3$$

(we have considered the uniform law of distribution of the absolute permeability over the thickness for the coefficient of variation $V = 0.55$). The modified permeabilities of the form (4) for water and oil, obtained according to the above-described algorithm, and the initial relative permeabilities for different seams are given in Fig. 2.

However in this work we propose another, more simple approach to construction of $K_w^{mod}(S)$ and $K_{oil}^{mod}(S)$, where

$$\overline{K}_w^{mod}(S) = \left[\frac{1}{H}\sum_{i=1}^{n} H_i K_w(S)\right] A(S),\quad \overline{K}_{oil}^{mod}(S) = \left[\frac{1}{H}\sum_{i=1}^{n} H_i K_{oil}(S)\right] B(S). \tag{5}$$

The dependences $\overline{K}_w^{mod}(S)$ and $\overline{K}_{oil}^{mod}(S)$ are obtained by correction of the permeabilities, average over the bed thickness, using the factors $A(S)$ and $B(S)$ which are constructed for the initial stratified bed. Figure 2 gives $\overline{K}_w^{mod}(S)$ and $\overline{K}_{oil}^{mod}(S)$, while Fig. 3 gives

$$\overline{\overline{K}}_w(S) = \frac{1}{H}\sum_{i=1}^{n} H_i K_w(S) \quad \text{and} \quad \overline{\overline{K}}_{oil}(S) = \frac{1}{H}\sum_{i=1}^{n} H_i K_{oil}(S).$$

For comparison we solved the one-dimensional and two-dimensional problems in the existing mathematical formulations [3] according to A. A. Samarskii's finite-difference schemes of the type of the alternate triangular method [4].

The results of the numerical hydrodynamic calculations for a one-dimensional displacement on the basis of the permeabilities (4) and (5) and a two-dimensional profile flow are given in Fig. 4 for the oil-recovery coefficient η and for the water fraction in the flow F as functions of the time of development of the bed t. Here A is the solution of the standard profile problem, B is the one-dimensional problem with the permeabilities (4), BC is the one-dimensional

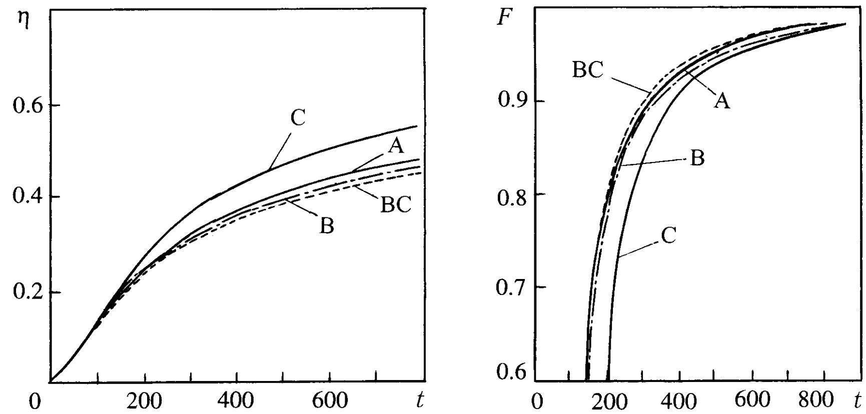


Fig. 4. Uniform law of distribution of $K(z)$ in seams ($V = 0.55$). η and F are dimensionless; t, days.

problem with the permeabilities (5), and C is the one-dimensional problem with the initial relative permeabilities averaged over the bed thickness. It is well seen that the calculations with the permeabilities (4) and (5) are close to the standard. Furthermore, the standard solution always lies between the one-dimensional solutions of the models C (upper bound) and B (or BC) (lower bound). The new permeabilities (4) and (5) obtained can be recommended for hydrodynamic calculations in stratified beds.

Let us now show the possibility of constructing $K_{\rm w}^{\rm mod}(S)$ and $K_{\rm w}^{\rm mod}(S)$ from formulas (3) and (4). In constructing for the aqueous phase, $\overline{K}_{\rm w}/K^* \geq 1$ must be preserved. According to formulas (1) we have $K_{\rm w}^{\rm mod}(S) = K_{\rm w}(S)\overline{K}_{\rm w}/K^*$. We consider the seam with the exponent $\alpha_i = \beta < \alpha$ for $\alpha = \alpha^*$. When $K_{\rm w}(S) = K_{\rm w0}[S_{\rm mov}(S)]^{\alpha}$ we have $K_{\rm w}^{\rm mod}(S) = K_{\rm w0}[S_{\rm mov}(S)]^{\beta}$, $\beta < \alpha$. Hence $K_{\rm w0}[S_{\rm mov}(S)]^{\alpha}\overline{K}_{\rm w}/K^* = K_{\rm w0}[S_{\rm mov}(S)]^{\beta}$ or $\overline{K}_{\rm w}/K^* = [S_{\rm mov}(S)]^{\beta-\alpha}$ but $[S_{\rm mov}(S)]^{\beta-\alpha} = \dfrac{1}{[S_{\rm mov}(S)]^{\alpha-\beta}} \geq 1$ since $[S_{\rm mov}(S)] \leq 1$ and $\alpha - \beta > 0$. Thus, we have obtained that for $\alpha > \beta$ we always have $\overline{K}_{\rm w}/K^* \geq 1$; therefore, such a construction is possible (analogously it is proved that $0 \leq \overline{K}_{\rm oil}/K^* \leq 1$).

The discrete series of distribution of K_i and H_i over seams is usually available is hydrodynamic calculations (otherwise, it is easy to pass to such a series when the continuous distribution is specified): K_i is the absolute permeability of the seam, H_i is the thickness of the seam, and $P(K_i) = H_i/H$ is its probability. If we want to represent a specific homogeneous seam with absolute permeability K_l and thickness H_l in the form of a fictitious stratified seam, it becomes necessary to solve the following system for the oil phase:

$$1 - \frac{S_i - S^*}{S^* - S_*} = \sum_{K_N \leq K_i} P(K_N)\,, \quad \frac{K_{\rm oil}(S_i)}{1 - S_{\rm mov}(S_i)}\,\frac{J(S_i)}{K_l} = K_{\rm oil}^*(S_i)\,, \quad J(S_i) = \sum_{K_N \leq K_i} K_N P(K_N)\,. \tag{6}$$

Such a system, even for the specified fixed number of seams of a fictitious stratified seam, has many solutions. If it is allowed that for these seams we have $H_1 = H_2 = \ldots = H_j = H_l/j$, where j is the number of seams ($j \geq 3$) (then $P_1 = P_2 = \ldots = 1/j$) and it is considered that their absolute permeabilities satisfy the inequality $K_1 < K_2 < \ldots < K_j$, then such a system has a single solution since it contains $2j$ linear equations and $2j$ unknown $S_1, S_2, \ldots, S_j$ and $K_1, K_2, \ldots, K_j$. Having solved it, we find the sought $K_1, K_2, \ldots, K_j$ which are required in constructing the fictitious stratified seam. In solving this problem for the aqueous phase, we use $\dfrac{K_{\rm w}(S_i)}{S_{\rm mov}(S_i)}\,\dfrac{K_l - J(S_i)}{K_l} = K_{\rm w}^*(S_i)$ instead of the second equation in (6); the system obtained also has a single solution.

Thus, for each homogeneous seam we can find the complex of H_i and K_i and represent it as the fictitions stratified seam. The modified permeability for a specific fictitious stratified seam can also be written in the following form: $K_w^{mod}(S) = K_{w0}[S_{mov}(S)]^{\alpha} \dfrac{K_l - J(S)}{S_{mov}(S)K_l}$; in the above considerations, $K_w^{mod}(S) = K_{w0}[S_{mov}(S)]^{\beta}$ for $\beta < \alpha$. By equating the right-hand sides of the last expressions we obtain $\dfrac{K_l - J(S)}{K_l} = [S_{mov}(S)]^{\beta+1-\alpha}$, where $J(S) = \int_{a_l}^{\bar{k}(s)} kf_l(k)dk$, $\bar{k} \le b_l$ for $K_l = \int_{a_l}^{b_l} kf_l(k)dk$. Therefore, for any S the inequality $0 \le [K_l - J(S)]/K_l \le 1$ must be fulfilled. Then $0 \le [S_{mov}(S)]^{\beta+1-\alpha} \le 1$. But $0 < S_{mov}(S) < 1$; therefore, $\beta + 1 - \alpha > 0$ and $\beta > \alpha - 1$. At the same time, the condition $\beta < \alpha$ is fulfilled; consequently, we have $\alpha - 1 < \beta < \alpha$.

Thus, with such a construction of $K_w^{mod}(S)$ the exponents for $K_w(S)$ in seams must differ by less than 1, i.e., $|\alpha - \beta| < 1$. With such a construction the modified permeability of the oil for a homogeneous seam of thickness H_q and with permeability K_q can be written in the form $K_{oil}^{mod}(S) = K_{oil0}[1 - S_{mov}(S)]^{\alpha} \dfrac{J(S)}{[1 - S_{mov}(S)]K_q}$. But for $\alpha < \beta$, $K_{oil}^{mod} = K_{oil0}[1 - S_{mov}(S)]^{\beta}$. By equating the right-hand sides of the last expressions we obtain $J((S)/K_q = [1 - S_{mov}(S)]^{\beta+1-\alpha}$. Since $0 < S_{mov}(S) < 1$ and $\beta - \alpha > 0$, we have $0 \le [1 - S_{mov}(S)]^{\beta+1-\alpha}$ and the inequality $0 \le J(S)/K_q \le 1$ will always be fulfilled. Consequently, with such a construction the condition $\alpha < \beta$ must be fulfilled for the oil phase and there are limitations on the difference in the exponents of $K_{oil}(S)$ in seams (as was the case for the aqueous phase).

NOTATION

$K_w(S)$ and $K_{oil}(S)$, relative phase permeabilities of the water and the oil respectively; K^*, average absolute permeability of the vertical section of the bed; $\bar{K}_w$ and $\bar{K}_{oil}$, average absolute permeabilities of the water and the oil; $K_w^{mod}(S)$ and $K_{oil}^{mod}(S)$, modified phase permeabilities of the water and the oil obtained by correcting the initial permeabilities $K_w(S)$ and $K_{oil}(S)$; $\bar{K}_w(S)$ and $\bar{K}_{oil}(S)$, permeabilities of the water and the oil averaged over the bed thickness; $\bar{K}_w^{mod}(S)$ and $\bar{K}_{oil}^{mod}(S)$, modified phase permeabilities of the water and the oil which are obtained by correcting the permeabilities of the water and the oil averaged over the bed thickness; $K_w^*(S)$ and $K_{oil}^*(S)$, relative phase permeabilities of the water and the oil, respectively, constructed for the fictitious stratified bed; V, coefficient of variation of the stratified inhomogeneity of the bed; α_i and β_i, exponents in the analytical dependences of the relative phase permeabilities of the seams; $A(S)$ and $B(S)$, correction factors used in constructing the modified permeabilities; $A^*(S)$ and $B^*(S)$, factors $A(S)$ and $B(S)$ for the fictitious stratified bed; H, bed thickness; P_i, probability of the ith seam; K_q, H_q, K_l, and H_l, absolute permeabilities and thicknesses of the specific homogeneous seams q and l; S, water saturation; S^*, maximum value of the water saturation (delivery gallery); S_*, minimum value of the water saturation; S_{mov}, "moving water." Superscript: mod, modified. Subscripts: w, water; oil, oil, mov, moving.

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